Revision Notes

Class - 10 Maths

Chapter 1 - Real Numbers

● **Real numbers:**

- All rational and irrational numbers taken together make the real numbers. On the number line, any real number can be plotted.

● **Euclid's Division Lemma:**

- A lemma is a verified statement that is utilised to prove another. Euclid's Division Lemma states that for any two integers a and b, there exists a unique pair of integers q and r such that $a = b \times q + r$ where 0 r < b ≤ .

- The lemma can be simply stated as : Dividend = Divisor × Quotient + Remainder

- For any pair of dividend and divisor, the quotient and remainder obtained are going to be unique.

● **Euclid's Division Algorithm:**

- An algorithm is a set of well-defined steps that describe how to solve a certain problem. The Highest Common Factor (HCF) of two positive integers is computed using Euclid's division algorithm.

- Follow the steps below to find the HCF of two positive integers, say c and d , with $c > d$.

Step 1: We apply Euclid's Division Lemma to find two integers q and r such that $c = d \times q + r$ where 0 r < d ≤ .

Step 2: If $r = 0$, the H.C.F is d , else, we apply Euclid's division Lemma to d (the divisor) and r (the remainder) to get another pair of quotient and remainder.

Step 3: Repeat Steps 1–3 until the remainder is zero. The needed HCF will be the divisor at the last step.

● **The Fundamental Theorem of Arithmetic:**

The process of expressing a natural number as a product of prime numbers is known as prime factorization. Apart from the sequence in which the prime components occur, the prime factorisation for a given number is unique.

Example: 12 2 2 3 = × × , here 12 is represented as a product of its prime factors 2 and 3.

● Finding LCM and HCF:

- HCF is the product of the smallest power of each common prime factor in the given numbers.

- LCM is the product of the greatest power of each prime factor, involved in the given numbers.

- For any two positive integers a and b, HCF a, b × LCM a, b = a × b ( ) ( )

- L.C.M can be used to find common occurrence sites. For instance, the time when two people running at different speeds meet, or the ringing of bells with various frequencies.

● Rational and Irrational numbers:

- If a number can be expressed in the form p /q where p and q are integers and q 0 ≠ , then it is called a rational number.

- If a number cannot be expressed in the form p /q where p and q are integers and q 0 ≠ , then it is called an irrational number.

● Number Theory:

- If p (a prime number) divides 2 a , then p divides a as well. For example, 3 divides 2 6 , resulting in 36 , implying that 3 divides 6. - The sum or difference of a rational and an irrational number is irrational

- A non-zero rational and irrational number's product and quotient are both irrational.

- p is irrational when p is a prime number.

For example, 7 is a prime number and 7 is irrational. The preceding statement can be proven by the process of “Proof by contradiction”.

● Decimal Expansions of Rational Numbers:

- Let p x = q be a rational number with the prime factorization n m 2 5 , where n and m are non-negative integers. The decimal expansion of x then comes to an end. Then x has a non-terminating repeated decimal expansion (recurring).

- If a b is a rational number, then its decimal expansion would terminate if both of the following conditions are satisfied :

a) The H.C.F of a and b is 1.

b) b can be expressed as a prime factorisation of 2 and 5 i.e in the form n m 2 5 where either m or n , or both can be zero.

- If the prime factorisation of b contains any number other than 2 or 5, then the decimal expansion of that number will be recurring.